

King surface brightness profile

1D surface brightness profile (as in Python's KingProjectedAnalytic1D) :

$$I(r) = A.r_c^2 \left(\frac{1}{\sqrt{(r^2+r_c^2)}} - \frac{1}{\sqrt{(r_t^2+r_c^2)}} \right)^2 \text{ (for } r < r_t \text{ only)}$$

where:

- r_c is the core radius
- r_t is the tidal radius or truncation radius ($I = 0$ for $r = r_t$)

The concentration index is $c = \log_{10}(r_t/r_c)$.

For the analytical calculations, see AL's black notebook on p.144-145 (16/3/2021).

I **define** : $x = r_c/r_t = 10^{-c}$.

$$I(x=0) = A. \left(1 - \frac{x}{\sqrt{1+x^2}} \right)^2$$

$$I(x=r_c) = A. \left(\frac{1}{\sqrt{2}} - \frac{x}{\sqrt{1+x^2}} \right)^2$$

The total flux, $F = F(r_t) = \int_0^{r_t} I(r).2\pi r, dr$, has the following expression:

$$F = A.r_c^2. \pi \underbrace{\left[\ln\left(\frac{1+x^2}{x^2}\right) - 4\frac{\sqrt{1+x^2}-x}{\sqrt{1+x^2}} + \frac{1}{1+x^2} \right]}_{\text{Ftotnorm, written } \Psi}$$

So : $A = \frac{F}{r_c^2 \Psi} = \frac{3631.10^{-0.4m}}{r_c^2 \Psi}$, in Jy/square arcsec if r_c is in arcsec.

$$A.r_c^2 = \frac{3631.10^{-0.4m}}{\Psi}$$

And the **surface brightness profile** for a given total AB magnitude m can be written:

$$\mu_{\text{King}}(r) = -2.5, \log_{10}(I) = m - 2.5 \log_{10}(3631.) + 2.5 \log_{10}(\Psi) - 5 \log_{10} \left[\frac{1}{\sqrt{r^2+r_c^2}} - \frac{1}{\sqrt{r_t^2+r_c^2}} \right]$$

In the above, all radii should be in arcsec so μ is in AB magnitudes per square arcsecond.

Ratio of the flux within r_c to the total flux (=flux in r_t) :

$$\frac{L_c}{L_t} = \frac{\ln(2) - 4x(\sqrt{2}-1)/\sqrt{1+x^2} + x^2/(1+x^2)}{\ln((1+x^2)/x^2) - 4(\sqrt{1+x^2}-x)/\sqrt{1+x^2} + 1/(1+x^2)}$$

I could not derive an analytical expression for the half-light radius r_h . Soeren provided the following approximate formula (from a fit) :

$$r_h/r_c = 0.547 * (r_t/r_c)^{0.486}$$

That works well enough (checked with numerical sums).